Transitional and Turbulent Pipeflow of Pseudoplastic Fluids

Richard W. Hanks* and Brad L. Ricks†
Brigham Young University, Provo, Utah

A modified mixing length model is developed which permits the computation of velocity distributions and frictional pressure losses for transitional and turbulent pipe flow of viscous, inelastic non-Newtonian fluids. The rheological model assumed is the empirical power law. The method represents an improvement over previous workers' results. The data of several authors, most of which could not be harmonized by previous models, are shown to be fully compatible with the present method. The Dodge-Metzner-Reed, "generalized Reynolds number" method of correlation is shown to be inappropriate for transitional and turbulent flow. It is further shown that acceptable models of transitional and turbulent flow must correctly account for rheological behavior of the fluid. The proposed method expressly excludes viscoelastic effects. The method is suitable for engineering pipeline design computations and requires only a knowledge of power-law index n to permit computation of velocity profiles and friction factors.

Introduction

THE importance of non-Newtonian flow in the chemical processing industries is well established. Much work, both experimental and analytical has been performed by many investigators in an effort to obtain both qualitative and quantitative descriptions of the many interesting and often puzzling phenomena associated with the flow of non-Newtonian materials. Dodge and Metzner,[1] Shaver and Merrill,[2] and Clapp[3] performed extensive experiments and reported conflicting empirical correlations of their data. Subsequent experimenters[4-8] have found that in addition to the non-Newtonian viscosity effect there exists also a separate drag reduction effect which has been ascribed by some to viscoelastic phenomena.

Various authors have attempted to account for these non-Newtonian effects in different ways. Recently Hecht[9,10] presented a numerical analysis of several sets of data in which he used a mixing-length model. In this analysis Hecht divided the flow into two regions, a wall region and a core region. In the wall region he used Van Driest's[11] modified mixing length expression while in the core region he used a constant value. In order to fit his derived friction factor equations to experimental data, Hecht found it necessary to allow the von Karman coefficient κ to assume a wide range of values, different from the usual value of 0.4. Although a wide variation in κ values was used to allow fitting of the data, Hecht did not present any correlation or method of determining the proper value to use. Thus, in effect, his results were essentially empirical.

The purpose of the present paper is to present an improved mixing-length model which permits accurate prediction of non-Newtonian pipe flow friction factors for both transitional and turbulent flow conditions. The present model is a modification of a previous one used successfully for Newtonian[12] and Bingham plastic[13] flows.

Analysis

For the steady axially symmetric flow of pseudoplastic non-Newtonian fluids in pipes a commonly used rheological model is the power law

$$\tau = K|\dot{\gamma}|^{n-1}\dot{\gamma} \tag{1}$$

where $n<1$ is a measure of the fluid's deviation from Newtonian ($n=1$) behavior, K is a viscous consistency factor (K = viscosity for a Newtonian fluid) and $\dot{\gamma} = -dv_z/dr$ is the shear rate. Equation (1) is empirical but widely used because of its simplicity.

Metzner and Reed[14] showed that by applying Eq. (1) on a point by point basis it could effectively be used to represent laminar flow friction factors if one used in place of n the n' slope of a Rabinowitsh-Mooney plot where n' is given by $d \ln \tau_w / d \ln (8\bar{v}/D)$. τ_w *is the wall shear stress,* $\bar{v}$ is the average velocity of flow and D is the pipe diameter. This technique has become widely acccpted[15] for engineering practice and is also assumed[1,14] to be valid for turbulent flow as well as laminar. This latter assumption is based primarily upon the correlation of Dodge and Metzner[1] who successfully employed the Metzner-Reed[14] method. However, Hanks and Christiansen[16] showed that the laminar-turbulent friction factors for non-Newtonian fluids having measurable yield stresses could not be correlated by the Metzner-Reed technique. These same data which could not be correlated by this method were later shown[17,18] to be well correlated by a Bingham plastic model. Therefore, it is not completely clear that the assumption of the validity of the Metzner-Reed technique for turbulent flow can be taken as well-established. This particular assumption will receive careful attention in the following treatment.

In the analysis which follows we shall deal only with viscous fluids whose rheology may be approximated by Eq. (1) and which have negligible elastic characteristics. These latter effects apparently account for large diviations from Newtonian behavior independently of non-Newtonian viscosity effects.[19]

The present analysis is based upon the mixing-length concept of Prandtl[20]

$$\tau = g(\dot{\gamma}) + \rho l^2 \dot{\gamma}^2 \tag{2}$$

where $g(\dot{\gamma})$ is the viscous rheological function (given by Eq. (1) in the present case), ρ is the fluid density and l is a mixing-length. Recognizing that $\tau = \xi\tau_w$ ($\xi = r/r_w$) and introducing the dimensionless variables $L = l/r_w$, $\Gamma = -du/d\xi$, $f = 2\tau_w/\rho\bar{v}^2$ (the familiar Fanning friction factor), $u = v/\bar{v}$, $\zeta = \Gamma/\Gamma_w$, one may rewrite Eq. (2) in the dimensionless form

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*Professor of Chemical Engineering.

†Engineer, Slurry Pipeline Division, Bechtel Inc., San Francisco, Calif.

$$\xi = \frac{K\bar{v}^n \Gamma_w^{\,n}}{\tau_w r_w^{\,n}} \zeta^n + \frac{2L^2\Gamma_w^{\,2}}{f} \zeta^2 \tag{3}$$

Since $\zeta = 1$, $L = 0$ when $\xi = 1$ it follows from Eq. (3) that

$$\Gamma_w = (r_w/\bar{v})\,(\tau_w/K)^{1/n} \tag{4}$$

With this simplification Eq. (3) becomes

$$\xi = \zeta^n + \tfrac{1}{8} R'^2 L^2 \zeta^2 \tag{5}$$

where

$$R' = [(3n+1)/n]\;[Re'\,(f/16)^{1-n/2}]^{1/n} \tag{6}$$

is a generalized turbulence parameter and Re' is the Metzner-Reed[14] generalized Reynolds number given for the power-law case by

$$Re' = 8\Big[\frac{n}{1+3n}\Big]^n \frac{\rho r_w^{\,n} \bar{v}^{2-n}}{K} \tag{7}$$

For the Newtonian limit $(n=1)$ R' becomes $Re(f)^{1/2}$, the parameter used previously[12] in an analysis of the corresponding problem for Newtonian fluids.

Eq. (5) is a dimensionless differential equation containing a parameter R'. The problem now is to integrate it. To do this one can use the identity

$$u = \int_\xi^1 \Big[-\frac{du}{d\xi'}\Big]\, d\xi' = \int_\xi^1 \Gamma(\xi')\, d\xi' \tag{8}$$

and the expression for the average flow

$$\bar{v} = \frac{2}{r_w^2} \int_0^{r_w} rv\, dr = 2\bar{v} \int_0^1 \xi \int_\xi^1 \Gamma(\xi')\, d\xi'\, d\xi \tag{9}$$

From Eq. (9) and the definition of ζ, we have

$$1 = 2\Gamma_w \int_0^1 \xi \int_\xi^1 \zeta(\xi')\, d\xi'\, d\xi \tag{10}$$

Upon integration by parts Eq. (10) becomes

$$1 = \Gamma_w \int_0^1 \xi^2 \zeta(\xi)\, d\xi \tag{11}$$

It is possible[21] to rearrange Eq. (4), (6), and (7) into the result

$$\Gamma_w = \Big[\Big(\frac{n}{1+3n}\Big)^n \frac{R'^2}{Re'}\Big]^{1/(2-n)} \tag{12}$$

Hence, we have from Eq. (11) and (12)

$$Re' = \Big(\frac{n}{1+3n}\Big)^n R'^2 \Big[\int_0^1 \xi^2 \zeta(\xi)\, d\xi\Big]^{2-n} \tag{13}$$

Once $L(\xi)$ is specified Eq. (5) defines $\zeta(\xi)$ implicitly for parametric values of R'. This equation can easily be solved numerically for $\zeta(\xi)$ and these values used in Eq. (13) to compute Re'. Finally, Eq. (6) is used to compute f. Thus, by selecting a series of values of R' one can generate an entire curve of f-Re'. The entire problem is seen to be one of numerical computation if a reasonable model can be developed for $L(\xi)$.

Mixing-Length Model

It has been shown experimentally[22,23] that the principal influence of polymer additives occurs very near the pipe wall. Therefore, a model which places principal emphasis on the near-wall region might be expected to be effective in describing non-Newtonian turbulent flow. Furthermore, since it is often true[23,24] that many non-Newtonian fluids exhibit extended ranges of transitional flow before full turbulence is observed, a reasonable model should also reflect the effect of transitional flow behavior.

Van Driest proposed[10] a modification of Prandtl's[20] basic mixing-length formulation which gives reasonable results in the near-wall region. Hanks proposed[12] a modification of Van Driest's model which also takes into account the effects of transitional flow characteristics. This model is

$$L = \kappa(1-\xi)(1-e^{-\phi(1-\xi)}) \tag{14}$$

where $\kappa = 0.36$ is the von Karman coefficient and

$$\phi = (R - R_c)/|\,(8)^{1/2} B \tag{15}$$

is a modified form of Van Driest's damping coefficient. R_c is the critical laminar-turbulent transition value of $R = Re\,(f)^{1/2}$. For Newtonian fluids $B = 22$ permits accurate representation of all available f-Re data.[12] For flow of Bingham plastic slurries Hanks and Dadia[13] showed that Eqs. (14) and (15) completely correlate all data if B is taken to be dependent upon a fluid/system parameter, the Hedstrom number. This latter parameter describes the deviation of the slurry from Newtonian behavior.

For the present case the model proposed for the mixing-length is Eq. (14) with ϕ replaced by ϕ' where

$$\phi' = (R' - R_c')/\text{-}8\text{-}^{1/2}\, B(n) \tag{16}$$

R_c' is the laminar-turbulent transition critical value of R' and is computed[21] from Hanks' transition theory.[25-27] According to that theory[25]

$$Re_c' = 6464 \frac{n}{(1+3n)^2} [n+2]^{(n+2)/(n+1)} \tag{17}$$

In terms of the definition being used for Re', it follows for laminar flow that $f = 16/Re'$. Thus, we may take $f_c = 16/Re_c'$, and from Eq. (6) it follows that

$$R_c' = [(3n+1)/n]\,(Re_c')^{1/2} \tag{18}$$

where Re_c' is given by Eq. (17). Figure 1 shows a plot of Eq. (18) in comparison with a number of experimental measurements taken from the works of the several authors cited in the Figure.

The question remaining to be settled is: what values shall be assigned to κ and B? Hecht[9] allowed κ to be variable and selected different values to fit experimental data. However, he offered no way to predict the dependence of κ on fluid or system parameters. In order to examine Hecht's method more thoroughly, several sets of data were analyzed[21] using $B = 22$ (the Newtonian value) and allowing κ to vary so as to fit the data. It was found that no reasonable correlation of κ with n was obtainable. Furthermore, it was found[21] that κ had to be variable with R' as well as with n. This was not thought to be reasonable. Consequently, κ was set at 0.36, the Newtonian value, for all subsequent calculations.

By analogy with the successful analysis of Hanks and Dadia[13] for flow of Bingham plastic fluids, it was felt $B = B(n)$ might allow correlation of the data. To test this hypothesis a number of data from several authors were fitted by the present model with B being variable. The results of these calculations are shown in Fig. 2 where B is represented as a function of n. The authors whose data were used are cited in the figure. The data in Fig. 2 are represented in the least squares sense[21] by the equation

$$B(n) = 22.4(n)^{-0.894} \tag{19}$$

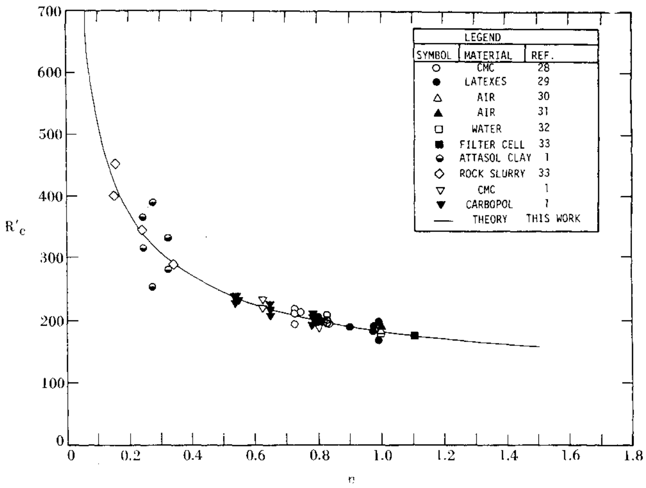

Fig. 1 Comparison of Eq. (18) with experimental data taken from the literature.

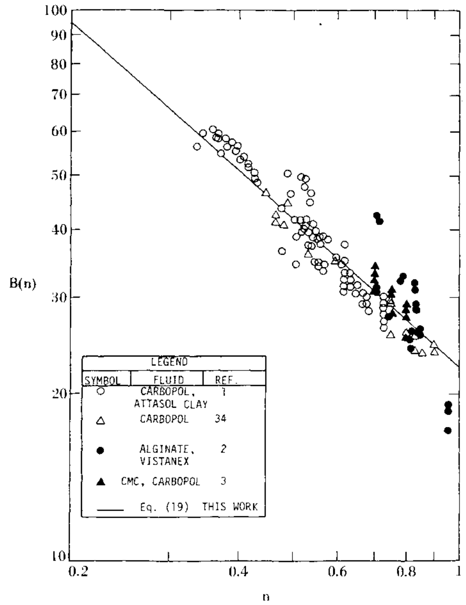

Fig. 2 Determination of dependence of $B(n)$ on n.

Actually, a negligibly small difference in the overall results[21] occurs if one uses the slightly simpler result $B(n) = 22/n$.

Calculation Method and Results

With the model which has been developed it is possible to calculate curves of f vs Re'. The procedure to be used is as follows. R_c' is computed from Eqs. (17) and (18). From Eq. (19), $B(n)$ is computed. Then, using Eqs. (14) and (16), together with the computed values of R_c' and $B(n)$, one Eq. (13) to compute Re' for a given value of R'. The evaluation of the integral in Eq. (13) requires solution of Eq. (5) to obtain $\zeta(\xi)$. Once Re' is calculated, one uses Eq. (6) to determine the corresponding value of f. If this process is repeated for a series of values of R', one can obtain a complete curve of f vs Re'.

This method of calculation was used to calculate complete curves for several authors' data. Figures 3 and 4 contain two of the sets of Carbopol data of Dodge.[1] Figures 5-7 show Clapp's friction data[3] in comparison with the present model. Figure 8 shows the data of Shaver and Merrill.[2]

From Eqs. (8) and (12) one may easily compute velocity profile curves for any given set of conditions. As an example

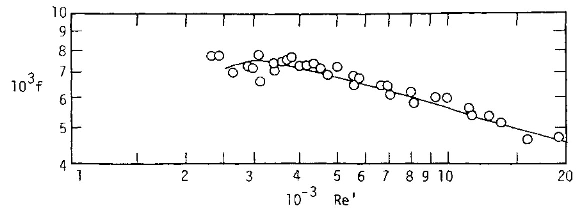

Fig. 3. Comparison of pipe flow data of Dodge[1] with theoretical model; 0.3% Carbopol solution, $n = 0.617$. Solid curve is theory.

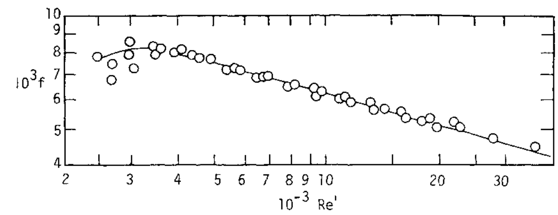

Fig. 4. Comparison of pipe flow data of Dodge[1] with theoretical model; 0.2% Carbopol solution, $n = 0.726$. Solid curve is theory.

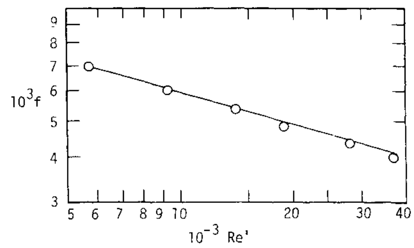

Fig. 5. Comparison of pipe flow data of Clapp[3] with theoretical model; $n = 0.70$ (data points calculated from Clapp's correlation). Solid curve is theory.

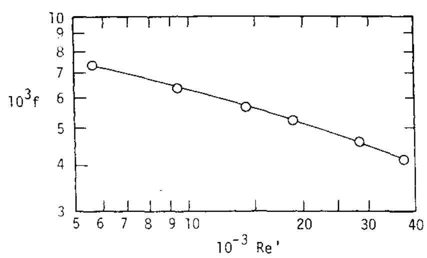

Fig.6. Comparison of pipe flow data of Clapp[3] with theoretical model; $n = 0.75$ (data points calculated from Clapp's correlation). Solid curve is theory.

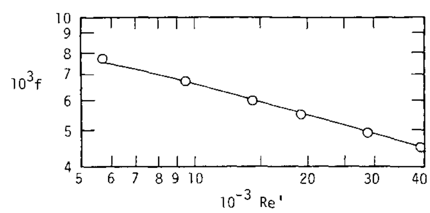

Fig. 7 Comparison of pipe flow data of Clapp[3] with theoretical model; $n = 0.80$ (data points calculated from Clapp's correlation). Solid curve is theory.

of this, curves were computed for Clapp's velocity profile conditions and are shown in comparison with his[3] data in Figure 9.

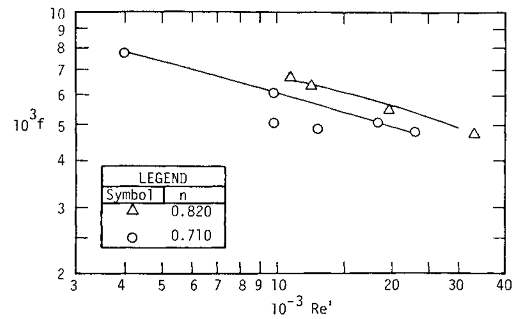

Fig. 8 Comparison of pipe flow data of Shaver and Merrill[2] with theoretical model; $n = 0.710$, 0.82. Solid curve is theory.

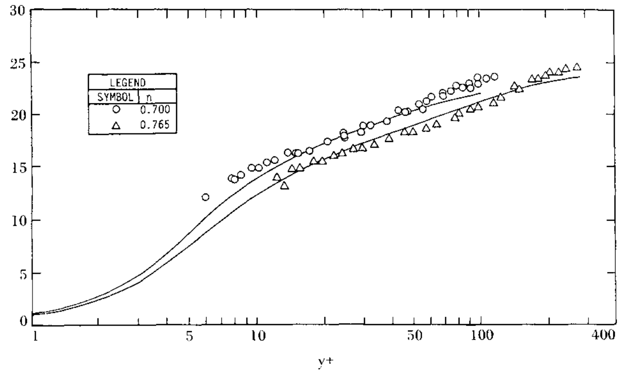

Fig. 9 Comparison of velocity profile data of Clapp[3] with theoretical model; $n = 0.700$, 0.765. Solid curves are theory.

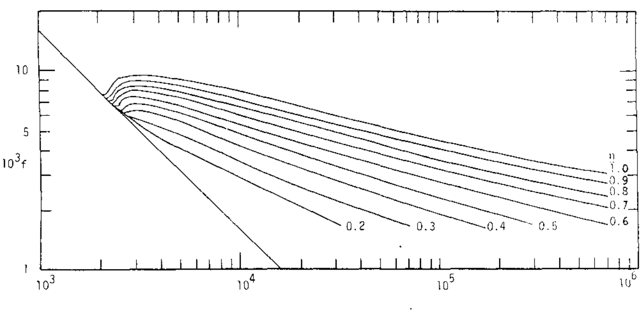

Fig.10 Design curves for turbulent pipe flow of power-law non-Newtonian fluids.

Figure 10 contains a set of design curves of f vs Re' for a series of values on n covering the full range of pseudoplastic values likely to be encountered in practice.

Discussion of Results

The model presented in the present paper is the first continuous velocity profile model which has been proposed for non-Newtonian pipe flow which is capable of accurately representing the available friction factor *and* velocity profile data for purely viscous non-Newtonian flow. In the figures shown above, the data of seven independent investigators have been well correlated. The data so correlated involve laminar-turbulent transition friction factors and Reynolds numbers, turbulent pipe-flow data, and velocity profile data for a variety of different fluids having a very wide range of non-Newtonian flow behavior indices. Such a range of correlation has not been previously accomplished.

It is evident from Fig. 1 that the R_c' data are completely correlated over the entire range of values of n likely to be encountered in practice. The curves in Figs. 3-8 are typical of the correlation obtained for turbulent pipe-flow friction data using this model. It is evident from these curves that the model adequately accounts for viscous non-Newtonian flow behavior.

In Fig. 9 the velocity profile curves have been expressed in terms of the variables u^+ and y^+. These variables are related to u and ξ by the relations

$$u^+ [u/(8)^{1/2}] \, [(Re'/R'^n)\,((1+3n)/n)^n]^{1/(2-n)} \tag{20}$$

$$y^+ = (R'/8^{1/2})^n \, (1-\xi)^n \tag{21}$$

which reduce to the familiar Newtonian universal variables when $n=1$. In physical terms $u^+ = v/v^*$ and $y^+ = \rho r_w{}^n v^{*2-n}/K$ where $v^* = (\tau_w/\rho)^{1/2} = \bar{v}(f/2)^{1/2}$ is the so-called "friction" velocity. It will be noted in Fig. 9 that in the region near $\xi = 0 (y^+ = y^+_{\max})$ the data lie consistently above the theoretical curve. This behavior in the central portions of the flow field is typical of Van Driest's model[10,12] for high Reynolds numbers.

At this point it is desirable to consider the following question. Metzner and Reed[14] proposed, and Dodge and Metzner,[1] and others[15] have perpetuated, the idea that any type of non-Newtonian fluid pipe-flow data could be correlated by use of a generalized Reynolds number defined by Eq. (7) with n and K replaced by n' and K', respectively. These latter parameters are, respectively, the slope and intercept of a straight line tangent to a plot of $\ln \tau_w$ vs $\ln (8\bar{v}/D)$. For a true power-law fluid $n' = n$, $K' = K$, and both are constants. For non-power law fluids, however, n' and K' vary with Re'. Metzner and Reed[14] successfully correlated laminar flow data by this technique. Dodge and Metzner[1] also successfully correlated Dodge's turbulent flow data s technique. However, Hanks and Christiansen[16] showed that the method failed to correlate laminar-turbulent transition data for fluids having yield stresses. Hanks and Pratt[18] subsequently showed that a proper rheological equation was required to permit correlation of Re_c data for fluids with yield stresses. Clapp[3] showed that the velocity profile expressions which Dodge and Metzner deduced[1] from their pipe-flow correlation (which was based upon this technique) were incompatible with his velocity profile data. Bogue and Metzner[34] also found it necessary to propose some complicated empirical correction factors which varied with both ξ and Re' in order to correlate their velocity profile data using the Dodge-Metzner-Reed generalized Reynolds number approach.

All of these observations lead one to question the validity of using the Metzner and Reed generalization in turbulent flow. In order to examine this question further, the data of Caldwell and Babbitt,[35] as analyzed by Metzner and Reed[14] were compared with the present model.The results of this comparison are shown in Fig. 11. It is very evident that this method fails to correlate the data. The same data were well correlated using a Bingham plastic model by Hanks and Dadia.[13] Fig.12 shows a set of thoria slurry data of Murdoch and Kearsey[36] in comparison with the present model using Metzner and Reed's n'. Again it is apparent that the method fails to correlate these data. Murdoch and Kearsey[36] found it necessary to employ a rheological equation due to Crowley and Kitzes[37] which incorporated both a yield stress and pseudoplastic behavior.

From the preceding results one may conclude that the use of the Dodge-Metzner-Reed generalization scheme in turbulent flow is not satisfactory as a general correlating method. Indeed, one may conclude that in order to correlate transitional and turbulent pipe-flow frictional resistance and velocity profile data for viscous non-Newtonian fluids wodel which accounts for the effects of both the viscous stresses and the turbulent Reynolds stresses in a continuous fashion, it is essential that a proper rheological model be used for the viscous part. This significant result is rather paradoxical in

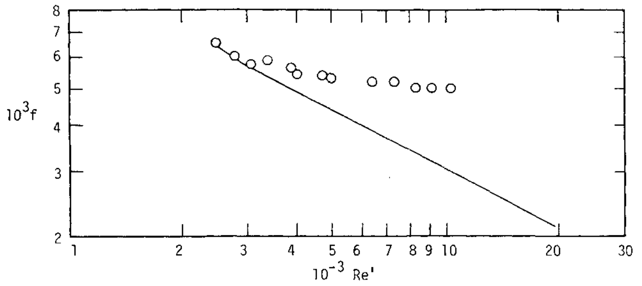

Fig.11 Data of Caldwell and Babbitt[35] for clay slurry; $n' \simeq 0.23$. Solid line is theory.

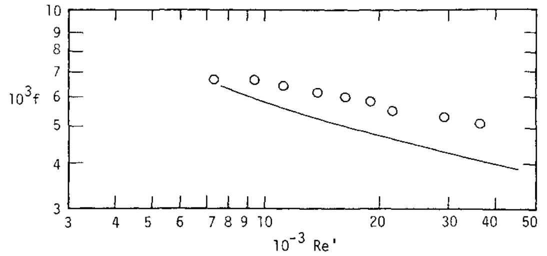

Fig. 12 Data of Murdoch and Kearsey[36] for thoria slurry; $n' \simeq 0.66$. Solid line is theory.

that for laminar flow one need not be so careful with the rheological equation (as witnessed by the success of the Metzner-Reed approximation) while in transitional and turbulent flow the proper formulation of the rheology is very important. Such a result could not have been intuitively forseen, and hence is not a trivial observation.

One final observation is in order. The above discussion of the need for proper rheological modeling in correlating transitional and turbulent flow data should make it very clear that any attempt to use a purely viscous model such as presented here to correlate data for drag-reducing viscoelastic fluids is doomed to failure. The present results clearly show that a large effect of non-Newtonian flow behavior can be well accounted for by proper rheological-turbulence modeling. Abundant literature data also clearly show that viscoelasticity effects are responsible for similar large deviations from Newtonian behavior. These effects must be accounted for separately from the non-Newtonian viscosity effects dealt with here. Consequently, the present work, while definitely useful and a marked improvement over any existing models, is by no means complete. It is anticipated that a modeling method similar to the present one but which deals only with viscoelastic effects will need to be developed. When such a model becomes available, it may then be feasible to combine the two models in some manner to produce a truly comprehensive model of transitional and turbulent flow of non-Newtonian-drag-reducing fluids. Work toward this end is presently underway.

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Engineering Notes

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Experimental Investigation of a Low Velocity Electrostatic Current Meter

James R. Williford* and David L. Murphree†
Mississippi State University, State College, Miss.

Introduction

THE present instrumentation state-of-the-art for measuring water movement includes hot film probes, Savonius rotor meters, electromagnetic meters, vortex shedding meters, and acoustic Doppler meters.[1] Whenever the temperature of the water varies between calibration and velocity measurement or during measurements, the influence of temperature on the hot film sensor must be considered. The rotating vane meter and vortex meter are insensitive to extremely low velocities, i.e., less than 0.1 fps. They are also especially inadequate for sensing small perturbations in ocean currents. The size and shape of the electromagnetic meter cause water distortion relative to its normally constant relationship to the freestream velocity, which may result in false measurements. The quality of the reflected signal and calibration difficulties are problems associated with the acoustic Doppler meters.

A need exists for more accurate measurements of the low velocity currents and small perturbations in ocean currents than can be obtained with the previously mentioned meters. These low velocities and small perturbations in the ocean currents have proved to be especially important near the ocean floor. These currents and perturbations have been instrumental to the build up or degradation of the ocean bottom, the migration of microscopic organisms, and the resulting migration of the larger forms of sea life which feed on them.

Electrostatic probes have been utilized successfully to measure the velocity of a flowing ionized gas.[2] This Note describes an experimental investigation of an electrostatic current meter which was found to be very sensitive to low velocities in sea water. The current meter demonstrates sensitivity to water velocities of less than 0.1 fps and small perturbations in the velocity. Although tests were not performed to directly measure the frequency response of the current meter, the meter did appear to exhibit a nearly instantaneous time response.

Theory of Operation

The investigated electrostatic current meter operates on the principle of electrolysis of sea water superimposed on the mass motion of the water past the meter. Every metal when immersed in an electrolytic solution and subjected to an increasing applied potential will conduct an almost constant current until a certain potential is reached.[3] At this potential, called the decomposition voltage, a sharp increase in the amount of current passed occurs. At the decomposition voltage steady electrolysis commences, and evolution of hydrogen and oxygen bubbles begins. It is above this potential that the current meter must be operated. Once above the decomposition voltage it is the superposition of the mass motion of the medium onto the normal electrolytic process that gives the current meter its sensitivity to the velocity of the sea water.

Obviously the meter is sensitive to the salinity of the water. However this should present no problem in the application of the device since it would be used in situ and the salinity of the site would be known. No investigation was made to determine the effect of temperature and pressure. These parameters will have an effect on the behavior of the meter,[4] however the effect may possibly be negated through the use of compensating methods similar to those used with hot wire anemometers.

Experimental Apparatus

To simulate sea water, a solution consisting of 35 parts of salt to 1000 parts of tap water by weight was used. To create the relative motion of the meter and water, a stationary channel of water with the meter mounted on a cart on a track was employed. The cart was moved along the track by means of a line attached from the cart to a spool attached to a system of gears driven by a low speed d.c. motor. With this system it was possible to measure very accurately the relative velocity between the meter and water by timing the motion between several reference points marked on the track.

The current meter geometry used for this experiment is shown in Fig. 1. It consists of two copper electrodes, one being number 26 wire and the other a flat disk of approximately ½ in. diam. The current meter was most sensitive when the wire was used as the negative electrode. Hence, this was the configuration used in the experiment.

Two different techniques of operating the electrostatic current meter were tested. The first method used was to apply

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*Research Engineer, Department of Aerophysics and Aerospace Engineering.

†Professor, Department of Aerophysics and Aerospace Engineering.